Phi Patterns in Nature and Beyond

Leigh Johnson, Donna Marie Pirich, Heather O'Connor, & Theresa Sampson

NY Omicron

St. Joseph's College
Patchogue, NY 11772

Presented at the 2009 National Convention.

Abstract

In his paper, "The Distance of the Planets from the Sun and Their Atmospheric Composition," Charles William Johnson postulates the existence of a Phi pattern in planetary orbits. The conjecture hinges upon the inclusion of Ceres as a dwarf planet. The author claims this inclusion is necessary in order to properly represent the asteroid belt between Mars and Jupiter, but fails to give a valid mathematical proof. We, the authors of this paper, investigate the validity of Johnson's work, and offer a mathematical proof based on regression analysis. Furthermore, we apply the same analysis to the lunar orbits of Neptune, Uranus, and Saturn, as well as the rings of Uranus. We believe this data analysis technique can also be used to predict the location of undiscovered moons in our solar system, as well as planets beyond Pluto.

1. Introduction

The existence of Phi patterns in nature has been a topic of great interest for mathematicians, beginning with Leonardo of Pisa (1170 – 1250), who first popularized the idea in a problem that he posed involving the growth of a hypothetical rabbit population (Burton 289-294). Phi is related to the Fibonacci sequence, $\{F_n\}$, where $F_1 = 1$, $F_2 = 1$, and $F_{n+2} = F_{n+1} + F_n$

for $n \geq 1$, and is defined as $\lim_{n\to\infty} \frac{F_{n+1}}{F_n}$ (Bicknell-Johnson). Hence, the Fibonacci numbers are:

$$\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987 \ldots\},$$

and Phi is an irrational number, which can be aproximated by 1.61803, correct to five decimal places. The figure below illustrates the rapid convergence of the sequence.

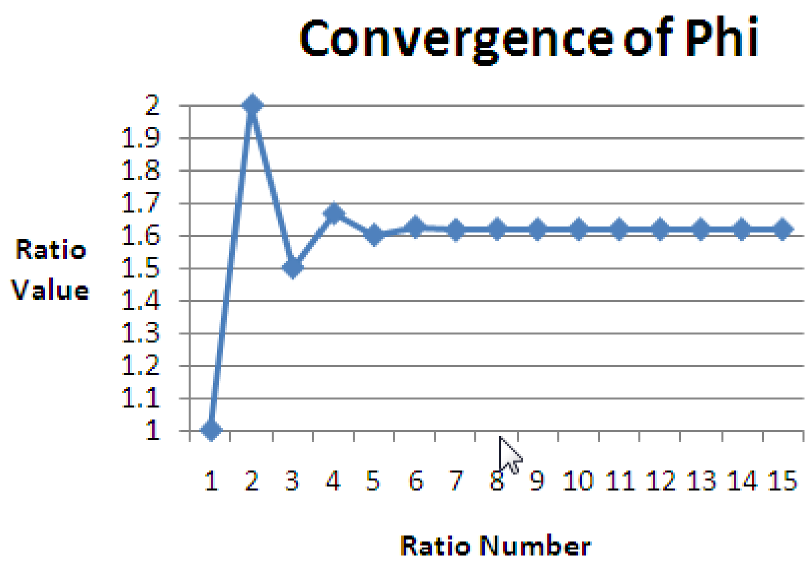

Figure 1

The Fibonacci sequence and the related Phi pattern have been observed throughout nature. The sprouting of new shoots during the growth process of various plants parallels the growth of Fibonacci's hypothetical rabbit population. This pattern can also be observed in the petals and seed heads of certain flowers, as well as in the spiral growth pattern of pinecones and nautilus shells (Knott).

For example, if one were to count the counterclockwise spirals created by the seeds of a sunflower, the number would be a Fibonacci number (Knott). Moreover, the number of clockwise spirals would be the previous Fibonacci number, and hence the ratio of these numbers is an approximation of Phi (Knott). As the sunflower grows, the approximation improves! The same phenomenon is also observed in pinecones (Knott).

2. Analysis of Planetary Data

The spiral pattern associated with Phi can also be observed in space. For example, the Milky Way is classified as a spiral galaxy (Morison and Penston 37). It appears to have a spiral pattern that resembles the growth pattern of pinecones, sunflowers, and nautilus shells (Harris). Therefore, the question arises as to whether a Phi pattern can be observed in our solar system.

Charles William Johnson postulates the existence of such a pattern in "The Distance of the Planets from the Sun and their Atmospheric Composition." The conjecture hinges upon the inclusion of Ceres as a dwarf planet. Johnson claims this inclusion is necessary in order to properly represent the asteroid belt between Mars and Jupiter, but fails to give a valid mathematical proof. Our research began with the development of such a proof, based on regression analysis. The result was a data analysis technique which we then applied to the lunar orbits of Neptune, Uranus, and Saturn, as well as the rings of Uranus.

We first analyzed the ratios of the distances from the sun of successive planets (normalized to Mercury), without including data on Ceres (see Table 1 and Figure 2). Using linear regression, with one-sigma error bars, we found Jupiter to be an outlier. (Note that since multiplication is commutative, dividing the distances in Column 2 of Table 1 by Mercury's distance from the sun, and then calculating successive ratios of distances, is equivalent to setting the first ratio in Column 3 to one, and calculating the remaining ratios directly from the planetary distances.)

Planet	Distance from Sun (km)	Ratio
Mercury	57, 900,000	1
Venus	108, 200,000	1.868739
Earth	149, 600,000	1.382625
Mars	227, 900,000	1.523396
Jupiter	778, 600,000	3.416411
Saturn	1, 433, 500,000	1.841125
Uranus	2, 872, 000,000	2.003488
Neptune	4, 485, 100,000	1.561664
Pluto	5, 870, 000,000	1.308778
	Mean	1.767358
	Std. Dev.	0.653183

Table 1

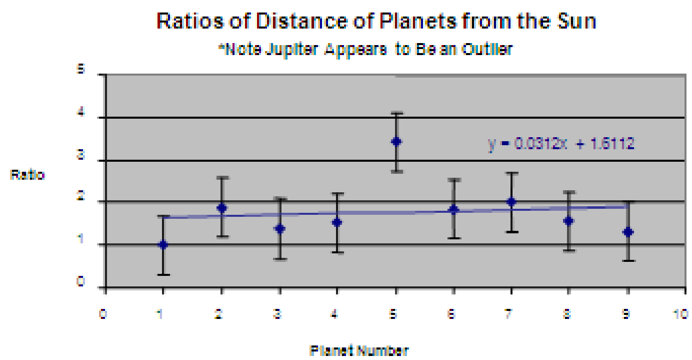

Figure 2

Using the linear regression equation established above,

$$y = .0312x + 1.6112,$$

we predict the location of a "missing planet" between Jupiter and Mars, recalculate the ratios of the distances of the planets from the Sun (normalized to Mercury), and finally establish a new regression line (see Table 2 and Figure 3).

Planet	Distance from Sun (km)	Ratio
Mercury	$57,900,000$	1
Venus	$108,200,000$	1.868739
Earth	$149,600,000$	1.382625
Mars	$227,900,000$	1.523396
Estimate	$402,744,880$	1.7672
Jupiter	$778,600,000$	1.933234
Saturn	$1,433,500,000$	1.841125
Uranus	$2,872,000,000$	2.003488
Neptune	$4,485,100,000$	1.561664
Pluto	$5,870,000,000$	1.308778
	Mean	1.619025
	Std. Dev.	0.3211

Table 2

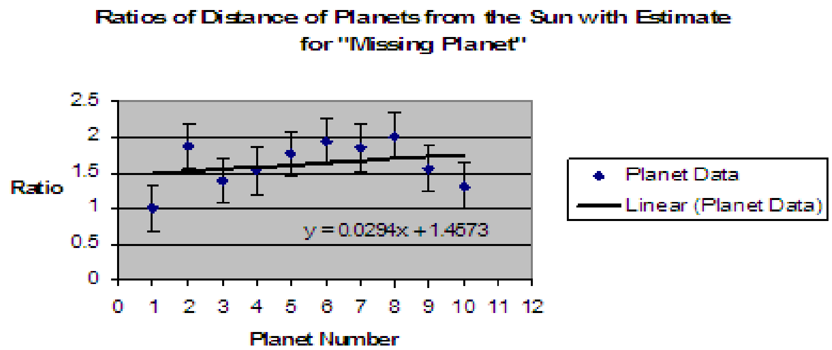

Figure 3

Inclusion of a "missing planet" resulted in a mean normalized planetary distance very close to Phi (1.619025). The location of the "missing planet" is within the vicinity of Ceres, thus justifying Johnson's inclusion of Ceres in his data analysis (see Figure 4).

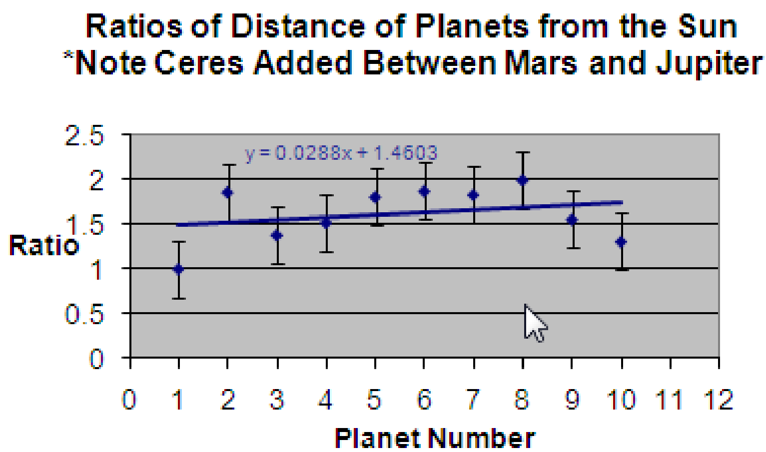

Figure 4

3. Analysis of Neptune

We began our study of lunar orbits with Neptune. The data pertaining to the thirteen known moons of Neptune (see Table 3) was collected by NASA (Williams, "Neptunian Satellite Fact Sheet"). Due to Neptune's great distance from Earth, and the limits of technology, discovery of these moons is relatively recent. Five of Neptune's moons were discovered in 2002 and 2003. As technology continues to improve, it is likely that still others will be found. In this section we will explore the possibility of the existence of a Phi pattern in the location of these moons. Our exploration was motivated by analogies that can be made between the Kuiper belt (which begins in the orbit of Neptune) and the asteroid belt between Mars and Jupiter discussed in the previous section.

Using the same approach as in the planetary data analysis, the distance between Neptune and its closest moon, Naiad, was taken to be the unit distance, and ratios of successive distances were calculated. The method of least squares was used to calculate the linear regression line determined by the moon numbers and corresponding distance ratios (normalized to Naiad). The results are plotted via Microsoft Excel, and one-sigma error bars are shown (see Figure 5). In the remainder of this section, we continue to iterate the technique until all outliers are eliminated, and predict the locations of possible undiscovered moons. Finally, the accuracy of our technique is analyzed by performing a regression analysis on the mean lunar distances resulting from the individual steps of the iterative technique.

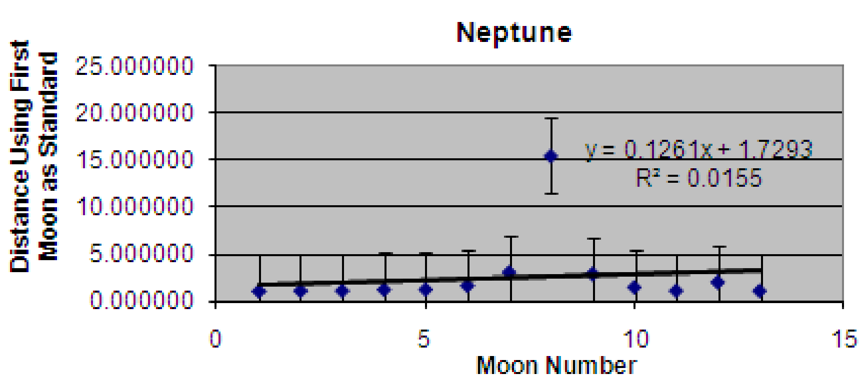

Figure 5

Moon	Moon #	Distance (km)	Ratio
Naiad	1	48,227	1.000000
Thalassa	2	50,075	1.038319
Despina	3	52,526	1.048947
Galatea	4	61,953	1.179473
Larissa	5	73,548	1.187158
Proteus	6	117,647	1.599595
Triton	7	354,760	3.015462
Nereid	8	5,513,400	15.541211
Halimede	9	15,730,000	2.853049
Psamathe	10	22,430,000	1.425938
Sao	11	46,700,000	1.050825
Laomedeia	12	46,700,000	1.981332
Neso	13	48,390,000	1.036188
		Mean	2.612115
		Std. Dev.	3.9447828

Table 3

Linear regression revealed that Nereid was an outlier. Using the regression equation

$$y = 0.1261x + 1.7293,$$

we postulate the existence of an undiscovered moon between Triton and Nereid (referred to as "Moon 1," in Table 4 below). We then recalculate the linear regression line and analyze the graph (see Figure 6).

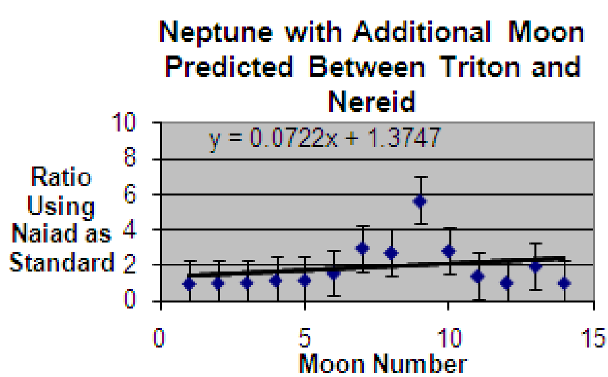

Figure 6

Moon	Moon #	Distance (km)	Ratio
Naiad	1	48,227	1.000000
Thalassa	2	50,075	1.038319
Despina	3	52,526	1.048947
Galatea	4	61,953	1.179473
Larissa	5	73,548	1.187158
Proteus	6	117,647	1.599595
Triton	7	354,760	3.015462
Moon 1	8	971,368	2.738100
Nereid	9	5,513,400	5.675911
Halimede	10	15,730,000	2.853049
Psamathe	11	22,430,000	1.425938
Sao	12	46,700,000	1.050825
Laomedeia	13	46,700,000	1.981332
Neso	14	48,390,000	1.036188
		Mean	2.612115
		Std. Dev.	3.9447828

Table 4

Nereid continues to be an outlier. Using the new regression equation, $y = 0.0722x + 1.3747$, we postulate the existence of a second undiscovered moon, "Moon 2," between "Moon 1" and Nereid (see Table 5). We then recalculate the linear regression line and analyze the graph (see Fig. 7).

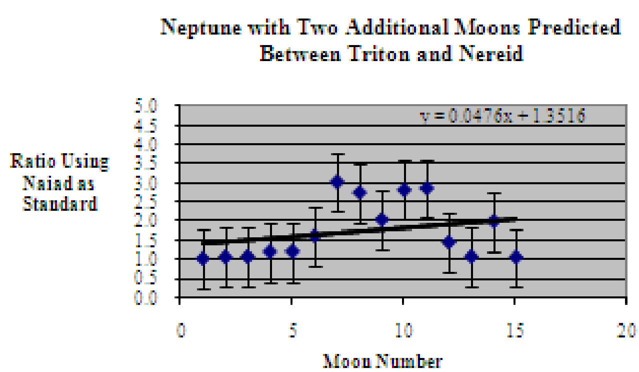

Figure 7

Moon	Moon #	Distance (km)	Ratio
Naiad	1	48,227	1.000000
Thalassa	2	50,075	1.038319
Despina	3	52,526	1.048947
Galatea	4	61,953	1.179473
Larissa	5	73,548	1.187158
Proteus	6	117,647	1.599595
Triton	7	354,760	3.015462
Moon 1	8	971,368	2.738100
Moon 2	9	1,966,535	2.024500
Nereid	10	5,513,400	2.803611
Halimede	11	15,730,000	2.853049
Psamathe	12	22,430,000	1.425938
Sao	13	46,700,000	1.050825
Laomedeia	14	46,700,000	1.981332
Neso	15	48,390,000	1.036188
		Mean	1.732166
		Std. Dev.	0.77263053

Table 5

After "Moon 2" is added, Triton, "Moon 1," Nereid, and Halimede appear to be outliers. Using the new regression equation, $y = 0.0476x + 1.3516$, we postulate the location and associated distance ratios of four more undiscovered moons (see Table 6). At this point, there appear to be no more outliers (see Figure 8). Hence, we end our predictions here, and evaluate the accuracy of the technique by performing a regression analysis on the iterative mean lunar distance data.

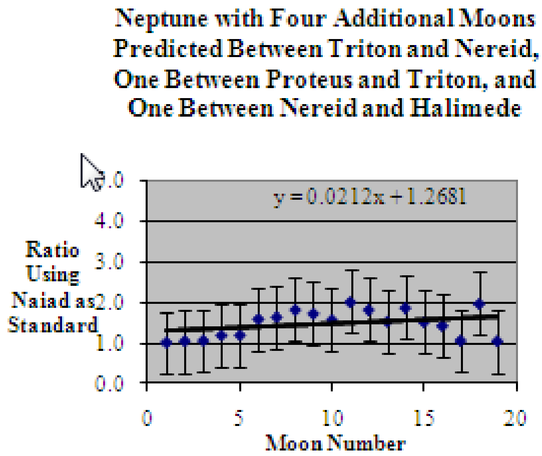

Figure 8

Moon	Moon #	Distance (km)	Ratio
Naiad	1	48,227	1.000000
Thalassa	2	50,075	1.038319
Despina	3	52,526	1.048947
Galatea	4	61,953	1.179473
Larissa	5	73,548	1.187158
Proteus	6	117,647	1.599595
Moon 3	7	193,976	1.648800
Triton	8	354,760	1.828883
Moon 4	9	614,586	1.732400
Moon 1	10	971,368	1.580524
Moon 2	11	1,966,535	2.024500
Moon 5	12	3,594,040	1.827600
Nereid	13	5,513,400	1.534040
Moon 6	14	10,338,728	1.8752
Halimede	15	15,730,000	1.521464
Psamathe	16	22,430,000	1.425938
Sao	17	46,700,000	1.050825
Laomedeia	18	46,700,000	1.981332
Neso	19	48,390,000	1.036188
		Mean	1.564620
		Std. Dev.	0.31763458

Table 6

Notice that Tables 3 through 6 show that with each iteration of our data analysis technique, the mean distance ratio for the moons of Neptune (normalized to Naiad) appears to be approaching Phi. Our results can be fit by a power regression curve, with correlation of 0.9931, which is quite accurate (see Table 7 and Figure 9).

Fibonacci #	Fibonacci Ratio	Mean of Mon Distance Ratio
1	1	2.612115
1	1	1.91645
2	2	1.732166
3	1.5	1.5642
5	1.666667	
8	1.6	
13	1.625	
21	1.615385	
34	1.619048	
55	1.617647	
89	1.618182	

Table 7

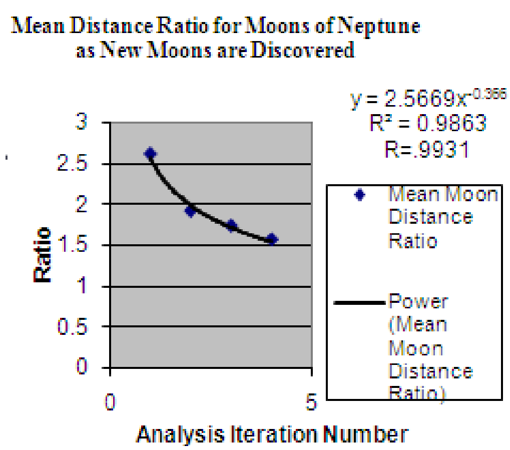

Figure 9

Note that Columns 1 and 2 of Table 7 illustrate the convergence of successive ratios of the Fibonacci numbers. Column 3 illustrates the mean lunar distance trend resulting from the four iterations of our regression analysis which were necessary to eliminate all lunar outliers. The ratios in Column 3 appear to be converging to a number which is close to Phi.

4. Analysis of Uranus

The next planet that was researched was Uranus, which has 5 major satellites and 22 minor satellites. The data pertaining to the moons of Uranus (see Table 8 at www.kappamuepsilon.org, the Kappa Mu Epsilon website) was collected by NASA (Williams, "Uranian Satellite Fact Sheet"). As in the planetary and Neptune data analyses, distances of satellites from Uranus were normalized to a unit distance equal to the distance between Uranus and its closest moon, Cordelia. The method of least squares was used to calculate the linear regression line determined by the moon numbers and corresponding distance ratios (normalized to Cordelia). The results are plotted via Microsoft Excel, and one-sigma error bars are shown (see Figure 10).

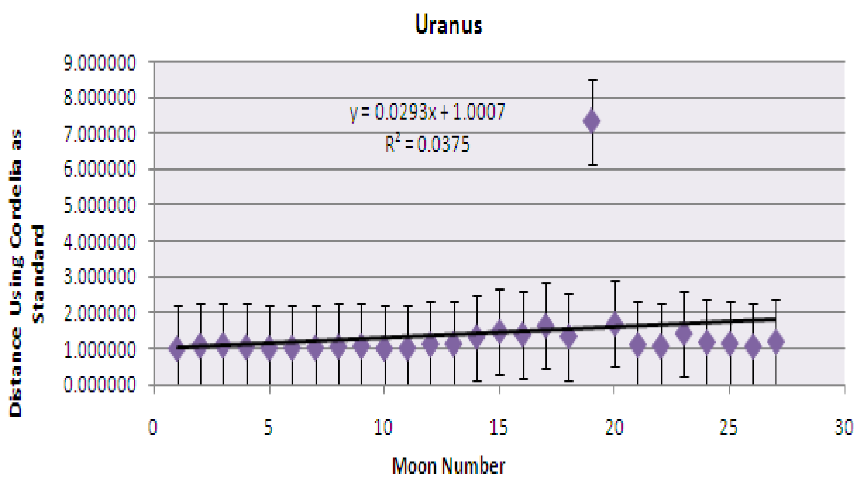

Figure 10

Calculation of the mean distance between Uranus and its moons did not reveal an observable connection to Phi. However, linear regression of the data shows a relationship between the distances, except for Francisco, which is an outlier. Adding possible undiscovered satellites, as was done in the case of Neptune (as well as the planetary data), would not be useful here due to the large number of minor moons, as well as the magnitude of the distance between these moons and the outlier, Francisco.

The moons of Uranus are classified as major or minor. The major moons are Miranda, Ariel, Umbriel, Titania, and Oberon. They are considered to be major moons because their radii are significantly larger than the radii of the minor moons (Williams, "Uranus Fact Sheet"). Due to their size, there is much more data available for the major moons. We decided to reanalyze our data using only the major moons to see if there was a Phi pattern, but we were unable to find one. In fact, without the minor moons, the mean is further from Phi.

We also analyzed the rings of Uranus, normalizing the radii of the rings to the radius of the equator of Uranus (see Table 9). Linear regression of the data reveals a strong correlation between ring number and normalized radius, and Phi lies within one standard deviation of the mean (see Figure 11).

Rings of Uranus	Distance (km)	Radius/Equator Radius
Equator of Uranus	25559	1
6	41837	1.636879377
5	42234	1.652412066
4	42571	1.665597246
Alpha	44718	1.749598967
Beta	45661	1.786493994
Eta	47176	1.845768614
Gamma	47627	1.863414062
Delta	48300	1.889745295
Lambda	50024	1.957197073
Epsilon	51149	2.00121288
	Mean	1.732021099
	Std. Dev.	0.271740742

Table 9

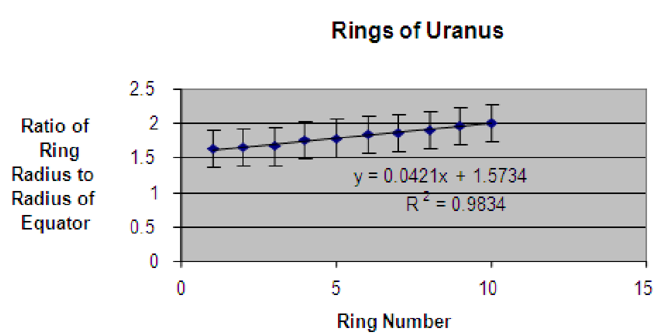

Figure 11

Further research uncovered many articles which discussed a relationship between the rings and moons of Uranus. The rings of Uranus are formed by dust particles released by surrounding moons (Goudarzi). These dust particles are formed when meteoric collisions occur with the moons. The dust particles become trapped in the lunar orbit by surrounding forces ("New Moons and Rings Found at Uranus"). In a 2007 MSNBC news release, "Planet Uranus Has a Rare Blue Ring," Goudarzi discusses the discovery of a rare Blue Ring about Uranus, and its relationship to the minor moon, Mab, which is believed to be the ring's "companion moon." According to the article, dust particles formed by meteoric collisions were released by Mab, and sent into the atmosphere to form this faint Blue Ring. The Blue Ring follows the orbit of Mab, and its blue color is due to the small size of the particles. On the other hand, the rings about Uranus which are predominantly red were formed by larger particles that reflect red light (Goudarzi). Further indications of a relationship between the moons and rings of Uranus were also noted in a 2005 press release, "New Moons and Rings Found at Uranus." A pair of rings and two new moons were discovered, due to the fact that one of the moons shared its orbit with a ring. This set of rings was so far from the rest, that they are considered to be their own system of rings.

The existence of a relationship between the rings and moons of Uranus led us to search for a Phi pattern based on this connection. Noticing that the mean of the ring data was an overestimate for Phi, while the mean of the moon data was an underestimate, we examined the average of these two estimates and found a much more accurate estimate for Phi:

$$\text{Mean Moon Distance from Uranus (normalized to Cordelia)} = 1.410173$$

$$\text{Mean Radius of Rings (normalized to the equator of Uranus)} = 1.7320210099$$

$$\text{Mean of Moon and Ring Data} = 1.57109705$$

The discovery of a Phi pattern which links the moons and rings of Uranus is not surprising. As discussed earlier, the rings are composed of particles from the moons, as in the small particles of the Blue Ring which are attributed to Mab (Goudarzi). Scientists have also found that the particles which comprise the rings of Uranus are being acted upon by surrounding forces which are influenced by the mass and orbit of the planet's satellites.

5. Analysis of Saturn

Finally, we investigated the moons of Saturn to determine whether or not they revealed a Phi pattern. Data pertaining to the moons of Saturn and their distances from Saturn was gathered from a NASA web site (Williams, "Saturnian Satellite Fact"). As of July 2007, sixty moons of Saturn have been identified. However, some of these discoveries are so recent that they are still unnamed. Using Johnson's study as a model, we set the distance of Pan (Saturn's nearest moon) from Saturn as the unit distance. We then calculated successive ratios of distances, as in the Fibonacci sequence. The data has been tabulated in Table 10, which can be found on the Kappa Mu Epsilon website, www.kappamuepsilon.org.

As in Johnson's work, we checked for the existence of a Phi pattern within these ratios, by computing the mean and standard deviation. The mean was calculated as 1.1246447, and the standard deviation was found to be 0.3698122. Figure 12 shows the results of a regression analysis of the raw data, along with error bars determined by the standard deviation.

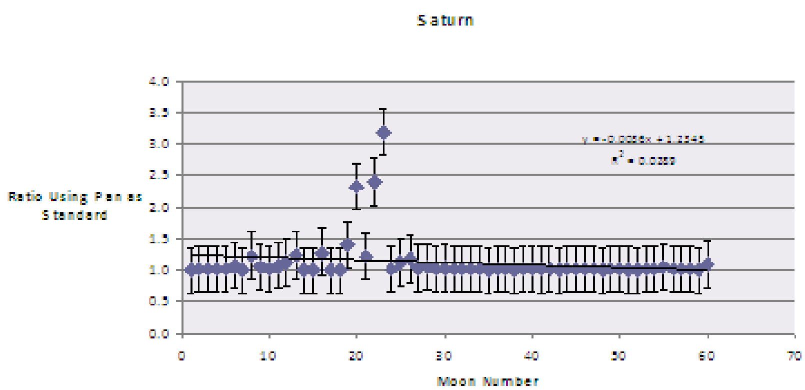

Figure 12

Titan, Iapetus, and Kiviuq appear to be outliers, and the data does not exhibit a readily identifiable Phi pattern. The average of the ratios is considerably less than Phi. In addition, two of the major moons of Saturn each have two Trojan moons which share the same orbit (Schombert). This forces the ratio unnaturally to one in the corresponding sequence of ratios. It was immediately apparent that the Trojan moons Calypso, Talesto, Helena, and Polydeuces, needed to be removed from the analysis. The list of data can be further reduced by considering only the most significant moons. Since the data was standardized to Pan, only those moons having mean density greater than or equal to that of Pan, 560km/m^3 , were considered (Williams, “Saturnian Satellite Fact”). This is justifiable since many of Saturn’s moons are actually large chunks that broke away from other moons (Schombert). For example, Hyperion is the largest irregular shaped moon observed, and is highly “pock-marked” (Schombert). This indicates that pieces of Hyperion broke away and entered alternate orbits. Another moon that shows signs of contributing to the formation of smaller moons is Mimas. This moon has a large crater that indicates it was struck by an asteroid or other cosmic object (Schombert). Therefore, the orbits of low density moons really depend on the original moons at the time of impact. Restricting the data in this way gives rise to the data in Table 11, below, as well as the regression analysis in Figure 13. One-sigma error bars enable us to identify outliers.

Moon	Moon #	Distance (km)	Ratio of Distance
Pan	1	133,583	1.000000000
Epimetheus	2	151,422	1.133542442
Janus	3	151,472	1.000330203
Mimas	4	185,520	1.224780818
Enceladus	5	238,020	1.282988357
Tethys	6	294,660	1.237963196
Dione	7	377,400	1.280798208
Rhea	8	527,040	1.396502385
Titan	9	1,221,830	2.318287037
Hyperion	10	1,481,100	1.212198096
Iapetus	11	3,561,300	2.404496658
Phoebe	12	12,944,000	3.634627804
		Mean Ratio	1.5938763
		Std. Dev.	0.7923295

Table 11

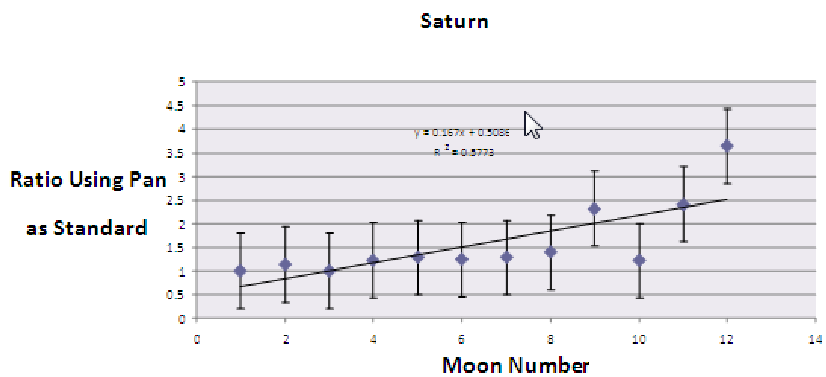

Figure 13

Saturn's moons suggest a strong correlation to Phi, similar to the pattern found in the planetary data. There were many factors to consider when analyzing the moons of Saturn, including the Trojan moons and the mean density of moons that were smaller than Pan.

6. Further Research

While working on any project one often wonders how the research can be extended in the future. One question that was raised by our research was whether or not a Phi pattern can be found in the moons of Jupiter. Considering the fact that Jupiter also has rings, if a Phi pattern was discovered, would it be similar to the pattern found in the moons of Saturn, or would it be more similar to the pattern exhibited by Uranus?

Another way our work can be extended is to draw a connection with Johannes Kepler's Laws of Planetary Motion. Kepler's Laws arose frequently in our research, as well as in discussions with mathematicians and scientists at various conferences. It would be interesting to see if our work is similar to Kepler's. For example, Kepler's Third Law states that $(\text{Period})^2 = (\text{Distance})^3$ (Morison and Penston 16), where the period is how long a planet takes to revolve around the Sun, and the distance is measured between the planet and the Sun. Using this information, Kepler knew where to look in the night sky for a particular planet. This is similar to our work, in that we found where a planet or moon should be using linear regression. Hopefully this project will lead others to discover new information about our universe.

Another avenue that this research could follow is an in-depth study of the asteroid belt between Mars and Jupiter. Could that have been a planet at one time? If so, what caused the planet's destruction? There has also been an asteroid belt discovered beyond Pluto, as well as a Plutiod. We would be very interested to know if that data would support the Phi pattern in our galaxy.

Finally, while researching the Milky Way, we learned about the classification of galaxies. For example, the Milky Way is a spiral class galaxy which exhibits a Phi pattern. Do other spiral class galaxies also exhibit a Phi pattern? Do the other two classifications, bar-spiral and elliptical, reveal a different pattern altogether, or none at all? We are sure that the quest for answers to these questions will lead to interesting discoveries in the future.

7. Conclusion

Further study of planetary data led to a data analysis technique based on linear regression which proved Johnson's postulated existence of a Phi pattern in the distance of the planets to the Sun (normalized to Mercury). This technique was applied to data collected on three planets in our solar system: Neptune, Uranus, and Saturn. It revealed a Phi pattern in all three cases.

An analysis of Neptune's satellite data led to the discovery of "missing" moons that fit a Phi pattern. This was similar to the pattern found by Johnson when he included Ceres (the largest asteroid in the asteroid belt between Mars and Jupiter) in his calculations. The mean distance between Neptune and its satellites (normalized to Naiad) was found to be close to Phi when "missing" moons were included in the analysis.

The search for a Phi pattern in the moon and ring data of Uranus proved to be tricky. Uranus has many moons, and our initial analysis produced a mean satellite distance that seemed too low for a Phi ratio. Therefore, linear regression would not correct this. Instead it would make the ratio even smaller. The ring data was taken into consideration, and at first it appeared that this data would not help our research, since it produced an estimate of Phi that was too high. However, information gained from the Voyager Mission revealed that the rings of Uranus did not form at the same time the moons did. The rings appear to be remnants of moons created prior to the rings, either broken up by a high-velocity impact or torn apart by gravitational effects. Therefore, a relationship exists between the satellite and ring data of Uranus. The average of the mean distance between the satellites and Uranus (normalized to Cordelia) and the mean of the radii of the rings of Uranus (normalized to its equator) resulted in an estimate of Phi.

Saturn's satellite data suggested that the mean ratio of the distance between Saturn and its moons (normalized to Pan) was an underestimate of Phi. There were many factors to consider when analyzing the moons of Saturn, including the Trojan moons and the mean density of moons that were smaller than Pan. Once these moons were removed from our analysis, a Phi pattern was revealed.

The results of our research uncovered further examples of Phi patterns in nature and beyond. These patterns are linked to the evolution of our solar system. We believe that similar patterns arise in other systems, and encourage future research in this area.

References

[1] M. Bicknell-Johnson, A Generalized Magic Trick from Fibonacci: Designer Decimals, *College Math. J.* **35** (2004) 125-126.

[2] R. Britt, "New Uranian Moon Found in Months-long Effort," `http://www.space.com/scienceastronomy/uranus_moon_021002.html`.

[3] D. Burton, "The Fibonacci Sequence.", *The History of Mathematics: An Introduction*, 6th ed., McGraw-Hill, 2005, 289-94.

[4] S. Gourdazi, "Planet Uranus Has a Rare Blue Ring," `http://www.msnbc.msn.com/id/12188326/`.

[5] C. Hamilton, "The Voyager Planetary Mission," `http://www.solarviews.com/eng/vgrfs.htm`.

[6] J. Harris, "The Fibonacci Series, Phi-Series and Synodic Mean," `http://www.spirasolaris.ca/sbb4d.html`.

[7] C. Johnson, "The Distance of the Planets from the Sun and Their Atmospheric Composition," `http://www.earthmatrix.com/extract62/mercury.html`.

[8] R. Johnson, *Miller & Freund's Probability and Statistics for Engineers*, 7th ed., Prentice Hall, 2005.

[9] R. Knott, "Fibonacci Numbers and Nature," University of Surrey, Department of Engineering & Physical Sciences, `http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html`.

[10] M. Livio, "Painters and Poets Have Equal License,", *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number*, Broadway Books, 2002, 159-200.

[11] I. Morison and M. Penston. *Pocket Guide to Stars and Planets*, New

Holland Publishers, 2005.

[12] "New Moons and Rings Found at Uranus," http://www.space.com/scienceastronomy/051222_uranus.html.

[13] J. Schombert, Astronomy 121: *The Formation and Evolution of the Solar System*: *Lecture 17: Moons*, Dept. of Physics, University of Oregon, http://abyss.uoregon.edu/~js/ast121/.

[14] D. Williams, "Neptunian Satellite Fact Sheet," http://nssdc.gsfc.nasa.gov/planetary/factsheet/neptuniansatfact.html.

[15] D. Williams, "Planetary Fact Sheet," . , http://nssdc.gsfc.nasa.gov/planetary/factsheet/index.html.

[16] D. Williams, "Saturnian Satellite Fact Sheet," http://nssdc.gsfc.nasa.gov/planetary/factsheet/saturniansatfact.html.

[17] D. Williams, "Uranian Satellite Fact Sheet," http://nssdc.gsfc.nasa.gov/planetary/factsheet/uraniansatfact.html.

[18] D. Williams, "Uranus Fact Sheet," http://nssdc.gsfc.nasa.gov/planetary/factsheet/uranusfact.html.

[19] D. Williams, "Uranus Rings Fact Sheet," http://nssdc.gsfc.nasa.gov/planetary/factsheet/uranringfact.html.